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Lattice of Quantum Predictions

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What is the structure of reality? Physics is supposed to answer this question, but a purely empiristic view is not sufficient to explain its ability to do so. Quantum mechanics has forced us to think more deeply about what a physical theory is. There are preconditions every physical theory must fulfill. It has to contain, e.g., rules for empirically testable predictions. Those preconditions give physics a structure that is "a priori" in the Kantian sense. An example is given how the lattice structure of quantum mechanics can be understood along these lines.

The term "lattice" in the title points to something mathematical, 'quantum' to something physical—so far I think I meet the criteria of our International Quantum Structures Association. Only "predictions" seems to indicate something quite different: It sounds not very physical, and much less mathematical.

I hope to show, though, that "prediction" is a central notion of physics, and so of all sciences; and that considering the structure of predictions can teach us a lot about mathematical structures.

1. PHILOSOPHICAL TRADITION

What we are looking for in our Association is what one could call a description of reality. All of us are working more or less on this subject, some more interested in mathematical structures, others more in the "physical" interpretation. Since I am, although a former physicist, now working in philosophy, my question has a slightly different shade. I am interested in the question

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How do we know what the structure of reality is?

The answer seems to be quite easy: Everybody knows that we know physics from experience. I.e., we first propose a theory, then we look into the experiments, and if that theory works—or, as long as it works—the theory is a true description of reality.

A more philosophically minded physicist might add—as we have learnt from Karl Popper—that, in a certain sense, every theory is only hypothetical: If a theory has always worked in the past, we believe that it will also work in the future. But there is no way to derive the general validity of a proposition logically from past instances; i.e., there is no way to *prove* a general proposition empirically. Popper says that we cannot *verify* a "lawlike" hypothesis empirically. We could at best imagine to *falsify* it: A law of nature must hold in *all* cases; thus, if we can find one single case where it does not hold, that law cannot be true. In real life it is not even possible to falsify a law of nature. For, in order to interpret a certain experiment as a counterexample of that law of nature, we have to presuppose the truth of many other laws of nature we use to describe the experiment; therefore, since those laws cannot be verified empirically either, it is likewise impossible to falsify a law of nature, in a strict logical sense.

Now this is not an invention by Popper. According to a biography of Galileo (Fölsing, 1983), already the Holy Inquisition argued in the same way against his empiricism: When he held that he could *prove* the Copernican theory to anybody who would look through his telescope, the inquisitors did not even take the trouble to look because they *knew* from good logic that is is impossible to prove a general proposition from a finite number of instances.

David Hume rediscovered the same, and his argument is more famous. Let me quote his central sentences (Hume, 1963, Sections IV, V):

> The contrary of every matter of fact is still possible; because it can never imply a contradiction, and is conceived by the mind with the same facility and distinctness, as if ever so conformable to reality. That the sun will not rise to-morrow is no less intelligible a proposition, and implies no more contradiction than the affirmation that it will rise. We should in vain, therefore, attempt to demonstrate the falsehood. Were it demonstratively false, it would imply a contradiction, and could never be distinctly conceived by the mind. (21) ... To say it is experimental, is begging the question. For all inferences from experience suppose, as their foundation, that the future will resemble the past, and that similar powers will be conjoined with similar sensible qualities. If there be any suspicion that the course of nature may change, and that the past may be no rule for the future, all experience becomes useless, and can give rise to no inference or conclusion. It is impossible, therefore, that any arguments from experience can prove this resemblance of the past to the future; since all these arguments are founded on the supposition of that resemblance. (32) ... All inferences of experience, therefore, are effects of custom, not of reasoning. (36) ... Belief is the true and propoer name of this feeling. (40)

Now the question of a logically certain proof of laws of nature does not really move any of us; no physicist would ever doubt that a physical theory that has worked in, say, 20 cases will work in all future, maybe several billion cases as well. But we could ask, and we should do so:

> *Why is that so? How can we be so sure that "the future will resemble the past"?*

The 18th century philosopher Imanuel Kant was the first one to ask this question seriously. His answer is quite surprising, even for us 20th century physicists. In some respects, certainly, Kant is old-fashioned: He did not know relativity or quantum theory, he considered that determinism and Euclidian geometry were self-evident. But still his basic ideas seem worth reconsideration, and this is what I would like to try, concerning our questions about the structure of fundamental physics. Kant says about his dependence on Hume: "I openly confess, the suggestion of David Hume was the very thing which many years ago first interrupted my dogmatic slumber, and gave my investigations in the field of speculative philosophy quite a new direction" (Kant, 1955, p. A13).

Kant's solution is his invention of "synthetic judgments *a priori.*" These are propositions which are not "analytic," i.e., not logically true, but which still can be proved without relying on special empirical evidence. (We know that any empirical evidence could not prove a general proposition, anyway). Such Kantian proofs are always based on the fact that the judgement in question is a "precondition for the possibility of any experience," as Kant calls it: Let us consider a proposition A for the rank of a "judgment *a priori.*" The argument to move A to this rank would be: "If A were not true, *no experience would be possible at all.*"

I cannot go into the details of Kantian philosophy; it is not necessary for this paper. But our excursion into the history of philosophy might make the following argument more plausible, as somewhat parallel with the Kantian argument.

2. MODERN A PRIORISM

As I said: We, as physicists, take empiricism as granted. But no doubt, we do make certain presuppositions that cannot be tested empirically. The principal one is our very program: We are looking for a physical theory. So we could reformulate the Kantian program, and say: "There may be certain preconditions for the possibility of any *physical theory*. Let us look for those!"—By the way, Kant modified his question in the same way: He looked for preconditions of any experience; but a precise formulation of experience would be a physical theory. So Kant actually dealt with Newtonian mechanics when he spoke more technically of experience.

So we begin with questions that sound much more down to earth than the Kantian ones:

What kind of collection of words and formulas are we prepared to accept as a physical theory?

This is certainly a question that cannot be decided experimentally, it is rather a question of our decision or of our consent on a program. But in deliberating this question we may arrive at some basic structures that hold for every physical theory, just because it is a physical theory.

I can very well understand the skeptism of any physicist when he hears of such a program. Errors are quite probable: Kant, e.g., considered Euclidian geometry was self-evident. I suppose that we, too, will make mistakes; we are working today, and our scientific grandchildren will discover things we have no idea of. But we are in a better position than Kant because we have a physics that is highly unified already. We have specimens of very general, very abstract theories that have a good chance to be *the* true and general theory of all reality. Thus we can start from a knowledge Kant could not have.

I am going to give, in the following, an example of an attempt at a justification of quantum mechanics *a priori* in the sense explained before. One might doubt that this makes sense, since we do *have* quantum mechanics, and we know that it is true. But I think this new aspect will in any case shed some new light on the structures we are looking at in our Association. And it might help us in our considerations about what will keep in quantum mechanics as it is now, and what is open to change in the course of further investigations.

3. QUANTUM MECHANICS A PRIORI

What do we expect from every theory of reality?

It must be an objective description, i.e., a collection of what we call laws of nature. A law of nature is *a general rule for empirically testable predictions*. I hope you will agree with that rather carefully composed definition: That a law of nature ought to be a general rule is self-evident; if the results obtained by that rule were not empirically testable it could not be a theory of reality, so this is clear, too. Only the last part of the definition could be doubtful: Is there a place for something as subjective as *predictions* in a physical theory?

Physicists normally do not like the idea that they deal with predictions. The time-dependent notion of prediction seems so subjective, as compared to the objectivity of the theoretical description. But what is an objective description? It is a description by a theory that can be checked, in principle, by anybody. How does he or she check the theory? By calculating the theoretical result and then measuring if it really comes out.

Thus the theory must *predict* a result in order that someone can check what comes out, *afterward*. The very objectivity of the theory is the reason that this theory is fundamentally about predictions.

So every law of nature is an interesting mixture of the temporal and the eternal: It deals with predictions, i.e., with something that depends extremely on time, the *now*, the observer, etc. But it deals with predictions in a way independent of time, being a law, a general rule, i.e, something that will never change and never depend on the now, the observer, etc.

So let us now look on what we have presupposed with that definition: "A law of nature is a general rule for empirically testable predictions."

It turns out that the concept of prediction is a key concept for the understanding of physics. The most general empirically testable prediction is a probability statement. I am not going to repeat the argument for this claim; see Drieschner (1992). I conclude: Every physical theory will give probabilities. Some theories might give only probabilities 1 or 0; we call those "classical." But we do not suppose that all systems are classical, i.e, we do admit indeterministic, i.e, quantum systems.

We are talking here of the intrinsic probability, not the probability that reflects only the lack of knowledge of the one has who makes the prediction. This is a difference we have learned to make only in learning quantum mechanics; it is the difference between the "pure case" and the "mixture." Abstractly one could say that the classical probability arises from a mixture of objectively different individuals, which have each quite definite but different properties. The new probabilities of the pure case arise even in an ensemble of exactly equal individuals, from an "objective" indeterminism; that is the specialty of quantum mechanics that makes it so interesting for philosophy.

One other thing we did learn from quantum mechanics is the abstract definition of a physical system. Classically there was no reason to distinguish between the everyday world and the physical systems within a physical theory. But it is already true in classical physics, what we discover anew in quantum mechanics: There is no concrete thing that is identical with a physical system. There is no such thing as a point mass. In defining a physical system we start from certain observables, e.g., position. In reality the position of a proton is the position of, e.g., a counter or a silver grain or a hydrogen bubble—or some other classically localized physical system.

We want to make predictions for such observables; and in order to be able to predict, we need the values of some observables at prior times. It turns out that there exist certain combinations of observables such that the knowledge of the values of those observables at one time enables a physicist to predict the values of the same observables at a later time. We call such a combination of observables a physical system. The simplest example is the classical point mass: If you know the position and momentum of (as you then say) a point mass, this is enough, in many cases, to predict the position and momentum (of the same point mass) for later times. So it is a good thing, in many cases, to take a point mass as a physical system, classically considered as an "idealization" of a concrete thing, e.g., an apple or a planet. A quantum mechanical system is composed of certain observables in the same way. The difference is that there is no concrete thing this physical system is an idealization of; there is no such thing as a concrete electron! To consider this insight, I think, would be helpful in a lot of disputes about the "reality" of microobjects.

We predict that one of those properties of which the physical system is composed will turn out true with a certain probability. Thus we distinguish between observables and states: Observables tell us which properties can be observed; the state gives us the probabilities for the observation of these properties.

In quantum mechanics it seems that by some odd chance some states coincide with properties, namely "pure" states with "atomic" properties. I shall try to show that this is not accidental, but that it cannot be different. This corresponds to the way Piron (1993) introduces the state.

Take any physical system with its observable properties. We compose the properties it has, because they give rise to good predictions—this is the reason of being for this sort of physical system. Thus we can suppose that for any future instant every property that belongs to that physical system is in some way predictable, i.e., it has a probability. Let me call this (generalized) probability distribution P. P gives a probability to every property of the physical system.

P is, on the other hand, a property of the physical system itself. This is doubted by many theoreticians of probability, because probability propositions seem to be applicable only to a whole lot of instances, and thus not applicable to a single physical system. The discussion of this question is rather complicated; I cannot give it here at length. Let me just give an outline: Probability predicts the relative frequency of the event in question for *any* number of experiments. Thus it applies not to a certain definite set of experiments, but to every set of a certain type of experiment, regardless of number or order—i.e., terminologically, to the *ensemble.*

Thus it seems evident: A property that applies in the same way to *any* set of experiments cannot depend on the set, but it can depend only on the type of experiment. So we are justified to conclude: Whenever we know that all the physical systems under consideration have the same properties (what we might call the "pure case"), probability is a property of the physical system itself. In this sense I agree with Popper's propensity interpretation of probability.

Thus P, the probability distribution for all observables, is a property of the physical system as well. We can even conclude that P is an atomic property. In order to give that argument we have to look first at the relation of *implication*.

We are dealing with two types of logic when we consider the structure of the properties of a physical system:

(i) The mathematics we are dealing with uses normal mathematical logic.

(ii) The propositions about the physical system have relations among each other that have also been called a logic, e.g., in the famous paper by Birkhoff and von Neumann (1936).

What I have to look at now is this "quantum logic." I am dealing with predictions; consequently the implication of this logic is an implication among predictions. Predictions are not true or false, as long as we do not presuppose determinism. Predictions are possible or impossible or necessary or probable with this and that probability. I propose the following interpretation, again not being able to explain it in detail: Prediction A implies prediction B means

Whenever A is necessary, B is necessary, too.

"Necessary" is equivalent, here, with "has probability 1," which, again, would deserve some careful consideration. This is what Piron calls a "true" proposition.

Now let us go back to the question of atomicity: Let us suppose that our probability distribution P, considered as a property of the system, is not an atom of the poset. Then there is an atom a, different from P, that implies P (we maintain that this is true of any prediction, due to the construction of physical systems); according to our consideration, this means

$$p(a) = 1 \;\rightarrow\; p(P) = 1$$

Now P contains a probability for a as for any prediction. This probability must be $p(a) = 1$, because P implies (in the mathematical sense) a probability for a,

$$P \;\rightarrow\; p(a) = x$$

Considering P as a property, i.e., as a prediction, this reads

$$p(P) = 1 \rightarrow p(a) = x$$

But $a \leq P$ (in the object language), which means

$$p(a) = 1 \rightarrow p(P) = 1$$

and thus by transitivity

$$p(a) = 1 \rightarrow p(a) = x$$

which means $x = 1$. Thus

$$p(P) = 1 \rightarrow p(a) = 1$$

i.e., $P \leq a$, P is equivalent with a, contrary to our assumption, thus P is an atom.

This justifies now what seemed to be rather accidental: that the states that do not depend on the lack of knowledge of the "observer," i.e., the pure states, correspond to atomic elements of the lattice of predictions itself. This justifies the "unital" states.

This is merely an example of what kind of argument can depend on our new way of asking, namely of asking for the structure of anything we would be prepared to call a physical theory; or—in the words of Kant—of asking for the "preconditions of the possibility of any experience" ("Bedingungen der Möglichkeit von Erfahrung überhaupt").

The discovery of quantum mechanics has forced us to think much more deeply about what a physical theory is, more deeply than at times where the impression prevailed that the (classical) theories give nothing but an image of what the world really is. I think that our analysis helps understand the problems of the interpretation of quantum mechanics, mainly those problems discussed under the label of "realism": I think we can see that the microobjects of quantum mechanics are not as real as—we are sure that—our everyday world is (Drieschner, 1992).

Naturally there are still unsolved problems, a lot of them. One problem I might get the reader interested in is the problem of the tensor product. I mean the following:

Apparently it is very practical that the stage on which quantum mechanics is played is a vector space. That makes it easy to write down the state space of a compound physical system, namely the tensor product of the state spaces of its components. For my *a priori* point of view this seems not only practical but even necessary: From our construction of physical systems it must always be possible to look at two independent systems as one compound system. In classical physics this is not very interesting, for all you get is the direct product of the two sets of properties. But in quantum mechanics this necessity seems to lead to a very serious constraint: The compound physical system has to be described by something that contains many direct products of sets of observable properties. Let us call such sets "alternatives."[2] The postulate is: The compound physical system has to be described by something that contains the direct product of every one of the "alternatives" of system 1 with every one of the "alternatives" of system 2; and the probabilities have to be, in addition, the products of the single-system probabilities.[3]

My question is: Are these constraints strong enough to exclude all quantum logics except the one we really have, the vector space lattice? Some work has been done on the question of the relation between orthomodular lattices and tensor products (e.g., Aerts and Daubechies, 1978; Matolcsi, 1975; Pulmannová, 1985; Zecca, 1981), but I do not think that there is an answer to exactly this question. So this rather nonmathematical paper might lead to some very mathematical problems.

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[2]"Alternatives" here means, following Scheibe *et al.* (1958), the set of nondegenerate eigenstates belonging to a set of compatible observables, corresponding to an orthonormal base in Hilbert space.

[3]Piron (1993) mentions this structure.